Complex roots of unity & polynomial equations

Exercise 3.10

Find all solutions to the equation $z^5 = 2 - 2i$, rounded to three digits.

We write $2 - 2i$ in polar form:

$$2 - 2i = 2\sqrt{2}\left(\cos\left(-\tfrac{\pi}{4}\right) + i\sin\left(-\tfrac{\pi}{4}\right)\right).$$

The five solutions have modulus

$$|z| = (2\sqrt{2})^{1/5} = 2^{3/10} \approx 1.231$$

and arguments

$$\theta_k = \frac{-\tfrac{\pi}{4} + 2\pi k}{5}, \qquad k = 0,1,2,3,4.$$

This gives:

$$z_0 \approx 1.216 - 0.193\,i$$
$$z_1 \approx 0.559 + 1.097\,i$$
$$z_2 \approx -0.870 + 0.870\,i$$
$$z_3 \approx -1.097 - 0.559\,i$$
$$z_4 \approx 0.193 - 1.216\,i$$